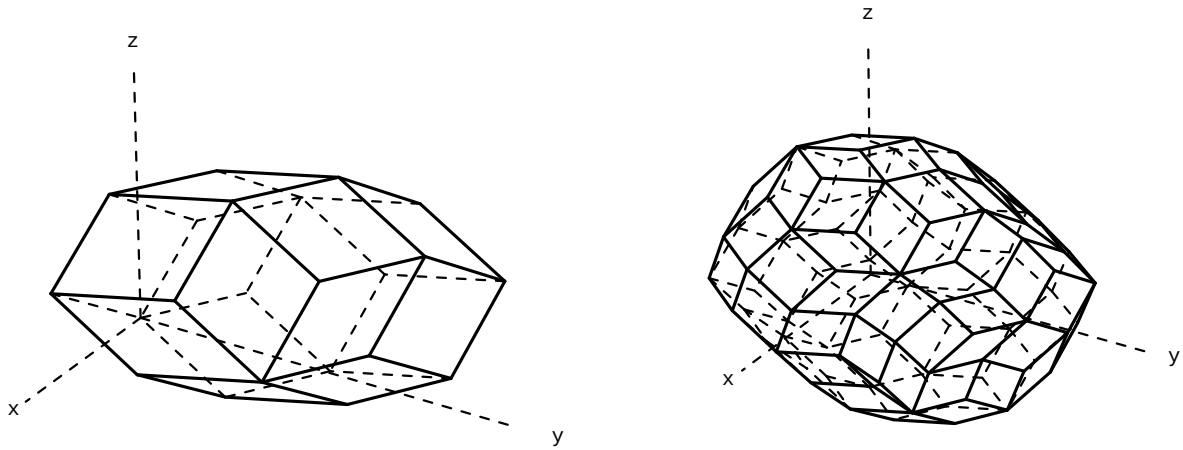
Recent Progress in Polyhedral Computation



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Recent Progress in Polyhedral Computation

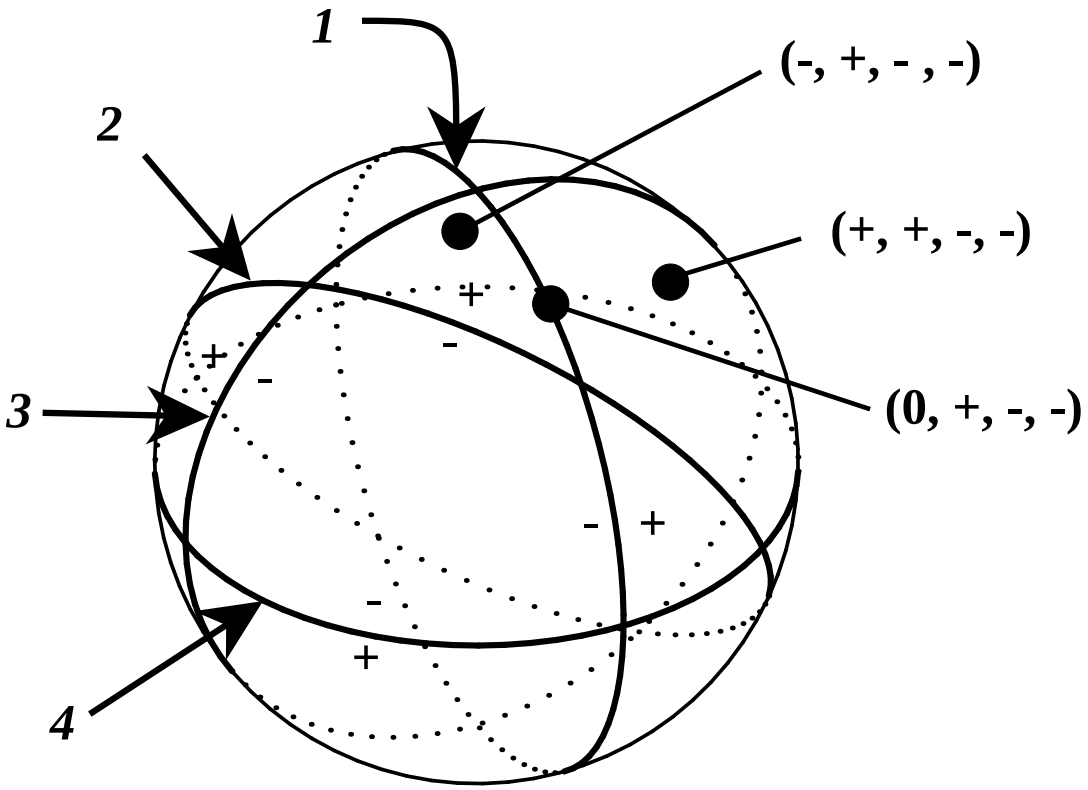
1. Polyhedral Representation Conversions
   - Algorithms and Complexities
   - Implementations
   - Related Problems
     – Polytope Volume, Redundancy Checking
2. Arrangement and Zonotope Construction
   - Algorithms and Complexities
   - Implementations
   - Application
     – Norm-Maximization over a Zonotope
     (with Liebling, Ferrez and Allemand)
3. Complete Catalog of Small Arrangements
   (with Finschi)
   - Algorithms
   - Catalogs of OMs and Arrangements

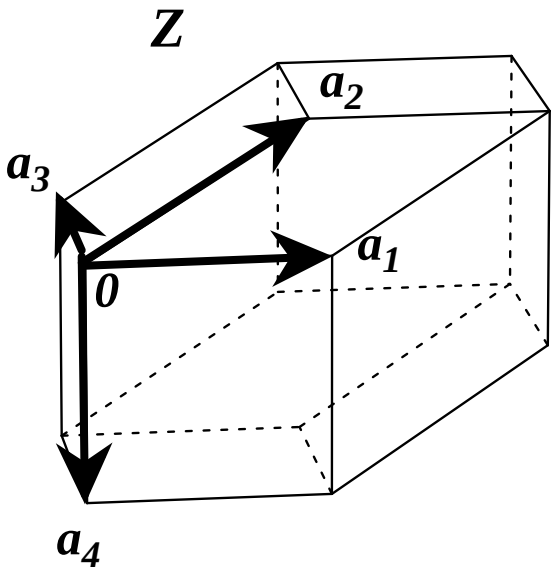
Arrangement Construction

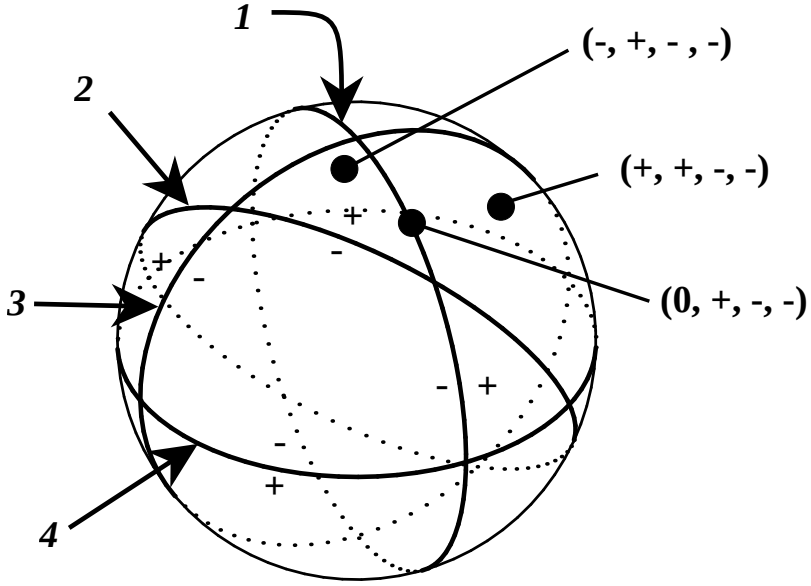
Problem:

Input: $A \in R^{m \times d}$.

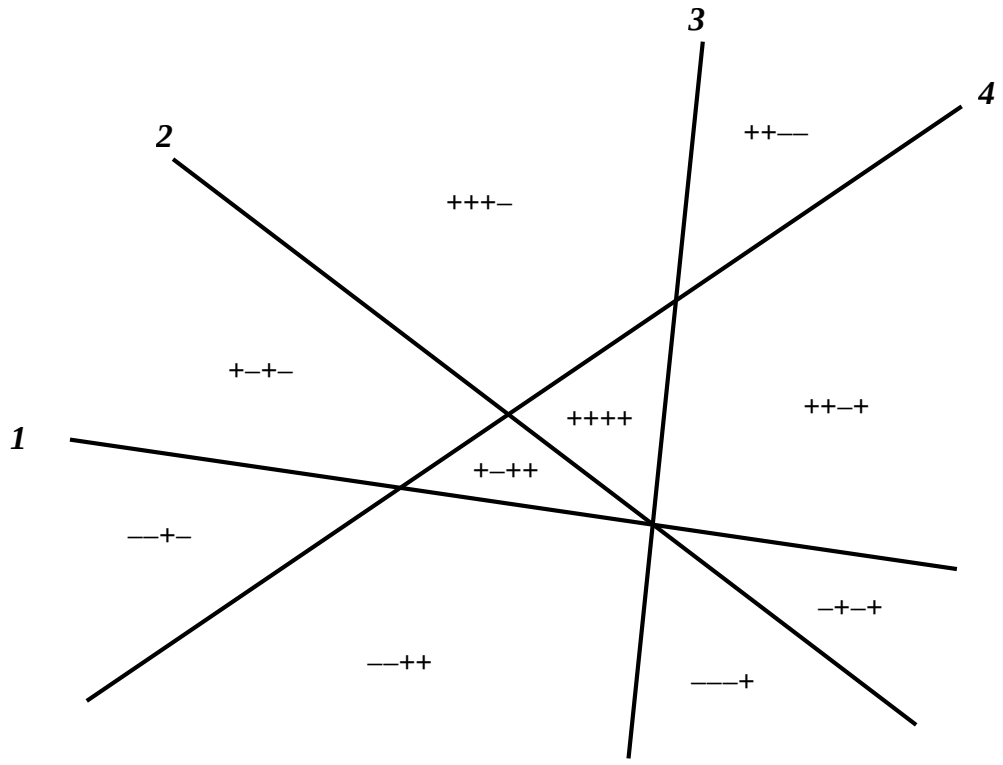
Output: All sign vectors of the column space
$C(A) := \{A\,x \mid x \in R^d\}$.




Z
a_2
a_3
a_1
0
a_4


1
2
3
4
(-, +, - , -)
(+, +, -, -)
(0, +, -, -)
+
-
+
-
-
+
-
+

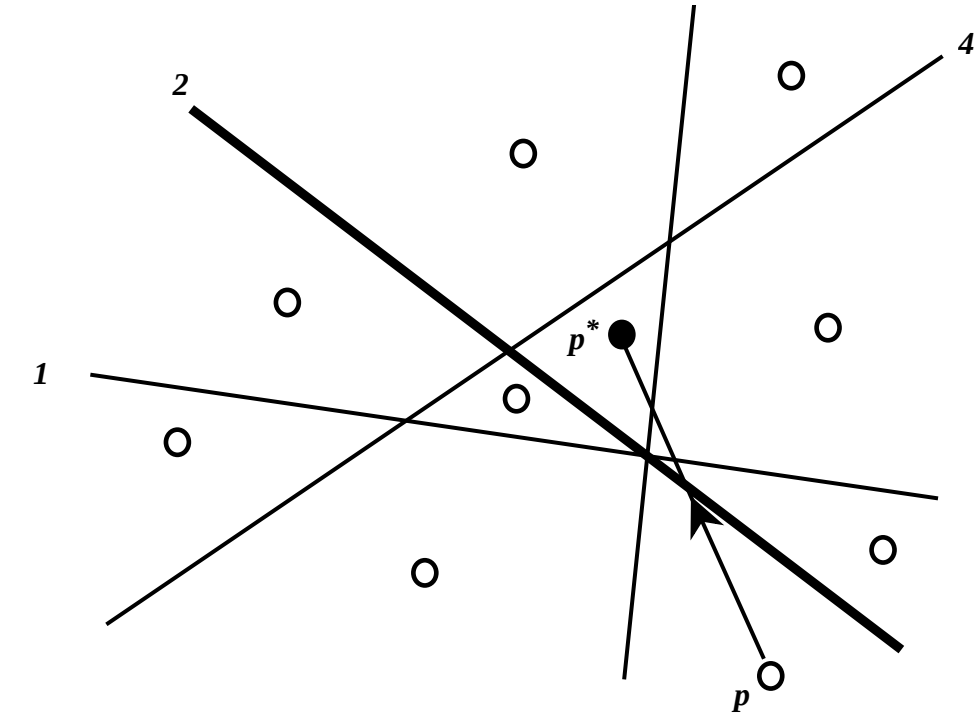
The Cell (Tope) Enumeration Problem

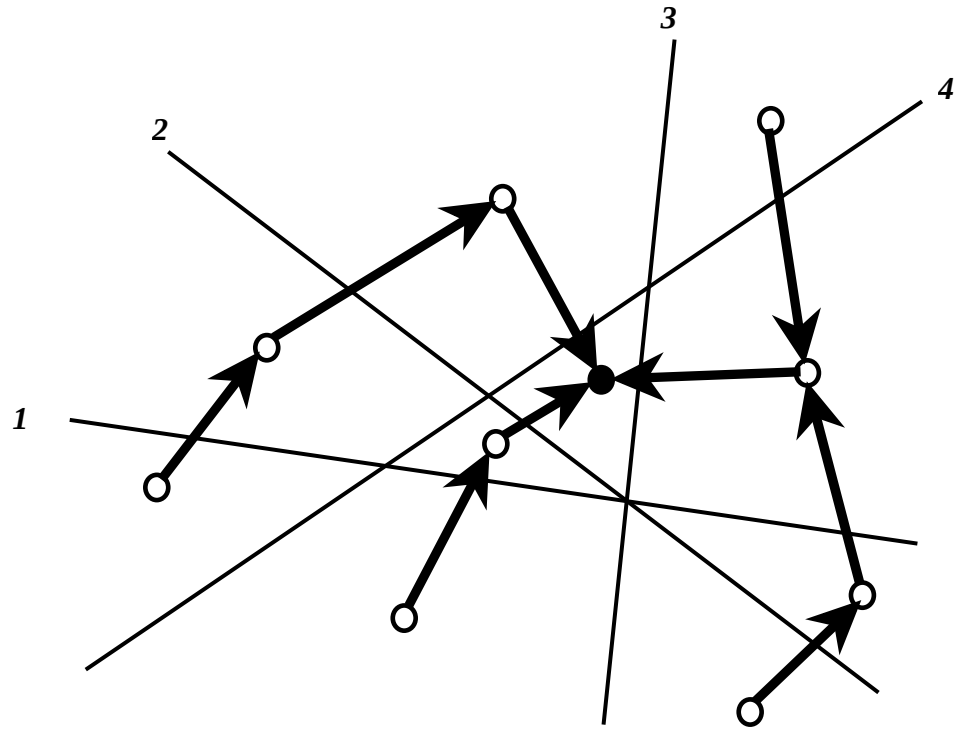


Theorem [Edelsbrunner et al]. For $d \geq 3$, there is an algorithm to generate all cells in $O(m^{d-1})$ time and $O(m^{d-1})$ space for fixed d.

Theorem [Reverse Search]. There is a reverse search algorithm of time complexity $O(m\ LP(m,d)\ c)$ and space complexity $O(m\ d)$ to generate all c cells.

Reverse Search Algorithm





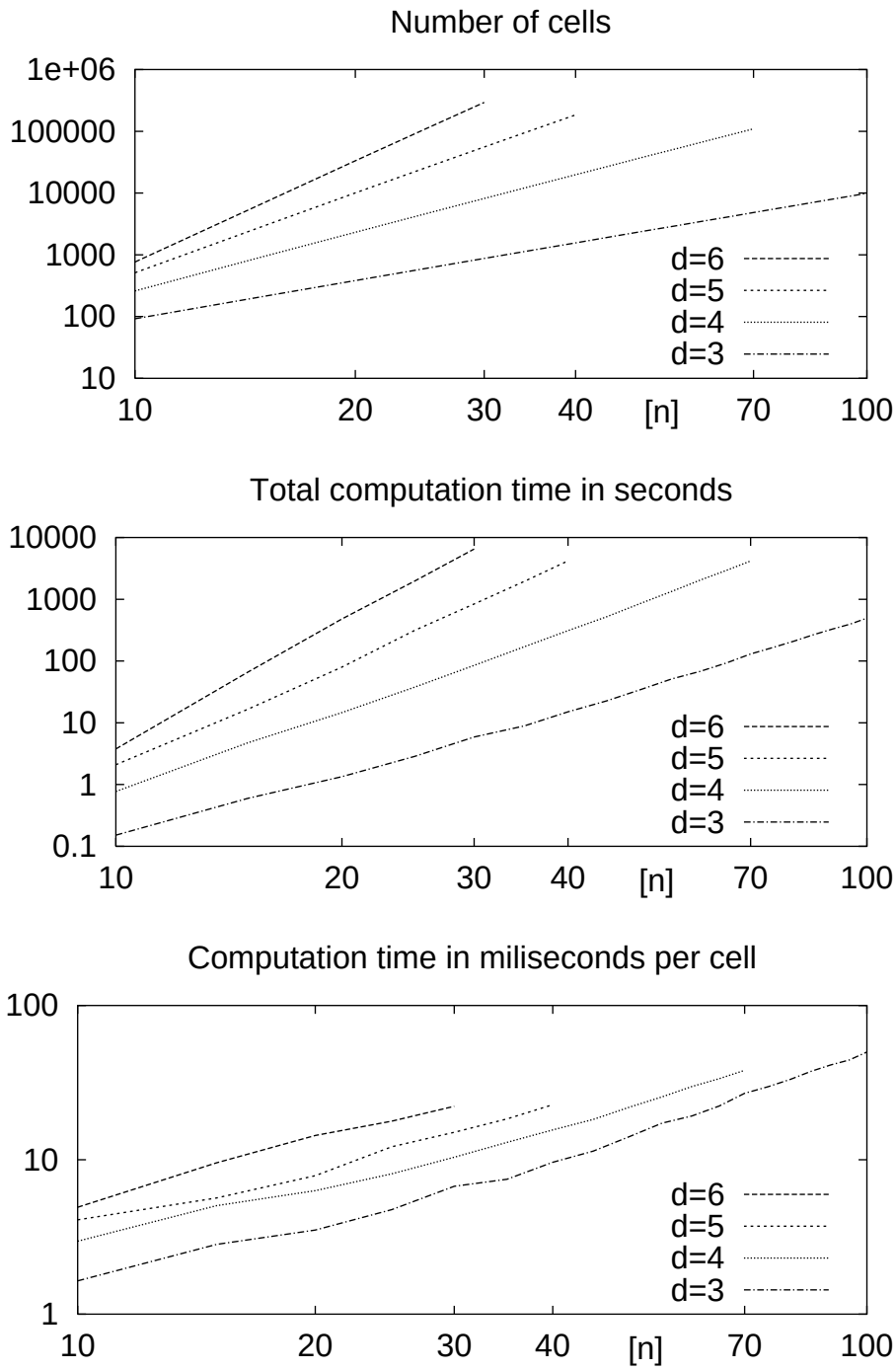


Figure 1: Number of cells, total computation time and time per cell for some small to medium problems.

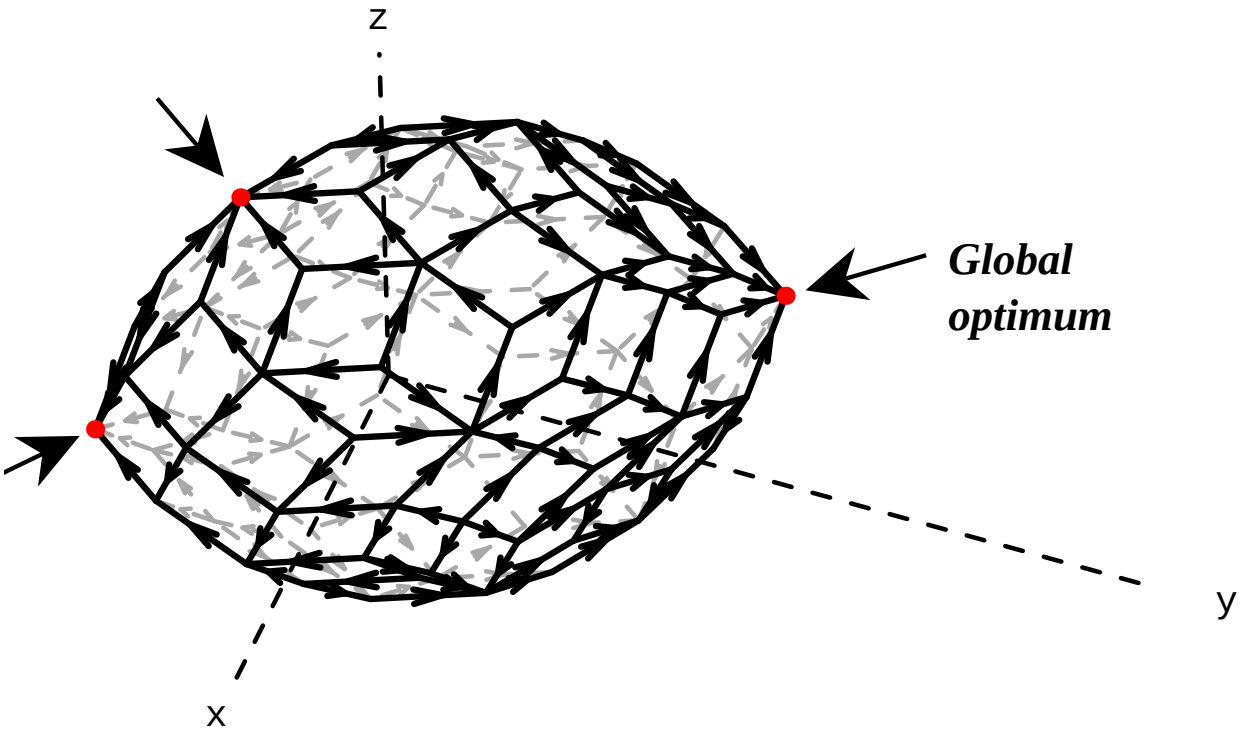


Figure 2: The three sinks of the norm-oriented graph of a 3-zonotope