3-1 Bibliography and Recent Advances

General references

1. D.Cox, J.Little, D.O'Shea, Ideals, Varieties, and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra, Undergraduate Texts in Mathematics, Springer, 2006. The chapters 1 and 2 are prerequisite of this tutorial.
2. D.Zeilberger, A Holonomic Systems Approache to Speical Function Identities, Journal of Computational and Applied Mathematics 32 (1990), 321–368. The original article is exciting!
3. T.Oaku, Algorithms for b-functions, restrictions, and algebraic local cohomology groups of D-modules, Advances in Applied Mathematics 19 (1997), 61–105. Finding a set of generators of a given integration ideal.
4. T.Oaku, Y.Shiraki, N.Takayama, Algebraic Algorithms for D-modules and Numerical Analysis, Z.M.Li, W.Sit (editors), Computer Mathematics, 23–39, World scientific, 2003. Holonomic (Schwartz) distribution.
5. M.Kauers, P.Paule, The concrete tetrahedron, Symbolic sums, recurrence equations, generating functions, asymptotic estimation, 2011, Springer. An introductory book on creative telescoping method.
6. T.Hibi edited. Gröbner Bases: Statistics and Software Systems, 2014, Springer. It includes an introduction to holonomic systems and algorithms for them with proofs.
7. F.Chyzak, The ABC of Creative Telescoping, Algorithms, Bounds, Complexity, 2014. A survey on the method of creative telescoping.

Fisher-Bingham distribution and its MLE

$$z(\Theta, \theta) = \int_{S^n} \exp(t^T \Theta t + \theta t)|dt| \tag{1}$$

$|dt|$: the Haar measure on the hypersphere S^n over which t runs. Θ: $(n+1) \times (n+1)$ real symmetric matrix. θ: real vector of the length $n+1$. Th The rank of differential equations is $2n+2$.

1. H.Nakayama, K.Nishiyama, M.Noro, K.Ohara, T.Sei, N.Takayama, A.Takemura, Holonomic Gradient Descent and its Application to the Fisher-Bingham Integral, Advances in Applied Mathematics 47 (2011), 639–658
2. T. Koyama, H. Nakayama, K. Nishiyama, N. Takayama, Holonomic Gradient Descent for the Fisher-Bingham Distribution on the n-Dimensional Sphere, arxiv:1201.3239, Computational Statistics (2013, online)
3. T.Sei, A.Kume, Calculating the normalising constant of the Bingham distribution on the sphere using the holonomic gradient method, Statistics and Computing (2013, online). [Degenerated cases need independent discussions.]

Fisher distribution and its maximal likelihood estimate (MLE)

X, Θ: 3×3 real matrices. $\Theta^\top$: the transpose of Θ. μ: the invariant measure on $SO(3)$.

$$z(\Theta) = \int_{SO(3)} \exp(\mathrm{Tr}\,(\Theta^\top X))\, d\mu(X).$$

Th The rank of differential equations is 4.

1. T.Sei, H.Shibata, A.Takemura, K.Ohara, N.Takayama, Properties and applications of Fisher distribution on the rotation group, Journal of Multivariate Analysis (2013) 440–455.

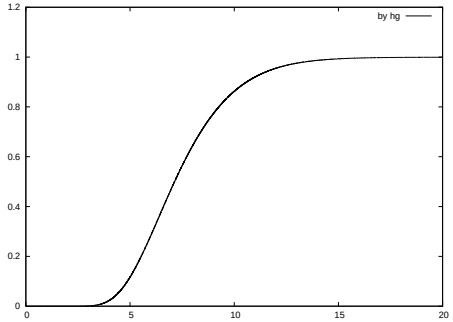
The cumulative distribution function for the first eigenvalue of Wishart matrices

Matrix hypergeometric integral ${}_1F_1$

X : $m \times m$ real matrix.

$$\int_{0<X<I_m} \exp(\mathrm{Tr}\, XY)|X|^{a-(m+1)/2}|I_m - X|^{c-a-(m+1)/2} dX,$$

$0 < X < I_m$ means that X and $I_m - X$ are positive definite symmetric matrix. $dX = \prod_{i \le j} dx_{ij}$. Th The rank of differential equations is 2^m.



$m = 10, n = 12, \beta = (1, 2, \ldots, 10)$

1. H.Hashiguchi, Y.Numata, N.Takayama, A.Takemura, Holonomic gradient method for the distribution function of the largest root of a Wishart matrix, Journal of Multivariate Analysis, 117, (2013) 296-312.

Orthant probability

$$z(\tau,\theta) = \int_0^\infty \cdots \int_0^\infty \exp\left(\sum_{i=1}^m \theta_i x_i + \sum_{i,j=1}^m x_i x_j \tau_{ij}\right) dx$$

$dx = dx_1 \cdots dx_m$. Th The rank of differential equations is 2^m.

1. L. Schläfli, On the multiple integral $\int^n dxdy \ldots dz$ whose limits are $p_1 = a_1 x + b_1 y + \cdots + h_1 z > 0, \quad p_2 > 0, \ldots, p_n > 0$ and $x^2 + y^2 + \cdots + z^2 < 1$, Quart. J. Pure Appl. Math. 2, 269–301; 3, 54–68, 97–108, 1858.
2. R.L. Plackett, A reduction formula for normal multivariate integrals, Biometrika, 41, No.3/4, 351–360, 1954.
3. H.I.Gassmann, Multivariate normal probabilities: implementing an old idea of Plackett's, Journal of Computational and Graphical Statistics, 12, No.3, 731–752, 2003.
4. T.Koyama, A.Takemura, Calculation of orthant probabilities by the holonomic gradient method, arxiv:1211.682.

A-distributions

$$z(\theta) = \int_C \exp(\sum_{j=1}^{n} \theta_j t^{a_j}) \prod_{i=1}^{d} t_i^{-b_i-1} dt, \ dt = dt_1 \cdots dt_d$$

$A = (a_{ij})$: $n \times d$ integer matrix. b_i: real number. a_j: the j-th column vector of the matrix A. $t^{a_j} = \prod_{i=1}^{d} t_i^{a_{ij}}$. Th The (holonomic) rank of the N.C. of the A-distribution associated to order polytopes has polynomial complexity.

1. T.Hibi, K.Nishiyama, N.Takayama, Pfaffian Systems of A-Hypergeometric Equations, arxiv:1212.6103 (v2 will be posted soon).

Mathematical byproduct: construction of a basis of twisted or rapid decay cohomology groups for complement of the zero set of a generic polynomial is reduced to a question in combinatorial commutative algebra.

More research topics will be in this conference!