微分方程式によるパラメトリック推定

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図, 資料など.

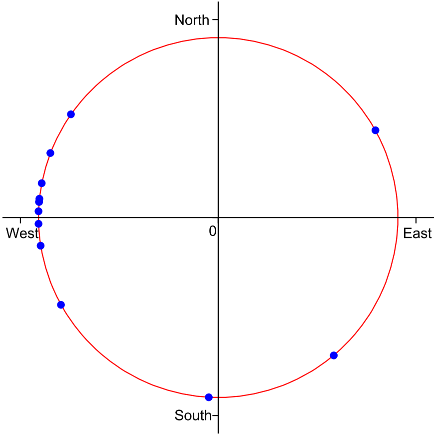

Figure: 札幌上空約１万メートル地点における午前９時の風向きの 13 日間のデータ (２０１１年１月１日から１４日 (11 日は欠損): 気象庁の気象統計情報から)

$(\cos t, \sin t)$ 達を上の 13 件のデータとする．HGD により

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A=test1(); isFit(A);
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$$\theta = (\theta_1, \theta_2) = (-0.1286, -1.5963).$$

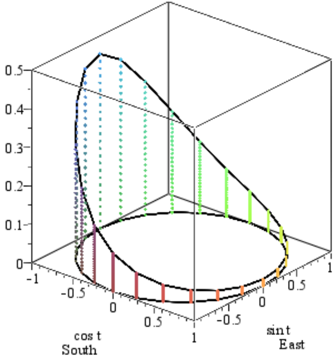


Figure: 推定したパラメータによる風向分布

Fisher-Bingham 分布

$$F(x,y,r) = \int_{S^n(r)} \exp(t^T x t + y t)|dt| \qquad (1)$$

$|dt|$ は球面上の Haar 測度. x は対称行列. y はベクトル.

Theorem

Fisher-Bingham 積分 $F(x,y,r)$ は x,y,r の *holonomic* 関数.

Theorem

付随する *holonomic* 系の *holonomic rank (number of the standard monomials)* は $2n+2$.
初期値計算を除く *HGD* が要する計算量は

$$O((2n+2)^3) \times \text{(steps of the convergence of the gradient descent)}$$

$n = 2.$

$$\partial_{x_{11}} - \partial_{y_1}^2, \partial_{x_{12}} - \partial_{y_1}\partial_{y_2}, \partial_{x_{13}} - \partial_{y_1}\partial_{y_3},$$
$$\partial_{x_{22}} - \partial_{y_2}^2, \partial_{x_{23}} - \partial_{y_2}\partial_{y_3}, \partial_{x_{33}} - \partial_{y_3}^2,$$
$$\partial_{x_{11}} + \partial_{x_{22}} + \partial_{x_{33}} - r^2,$$
$$x_{12}\partial_{x_{11}} + 2(x_{22} - x_{11})\partial_{x_{12}} - x_{12}\partial_{x_{22}} + x_{23}\partial_{x_{13}} - x_{13}\partial_{x_{23}} + y_2\partial_{y_1} - y_1\partial$$
$$x_{13}\partial_{x_{11}} + 2(x_{33} - x_{11})\partial_{x_{13}} - x_{13}\partial_{x_{33}} + x_{23}\partial_{x_{12}} - x_{12}\partial_{x_{23}} + y_3\partial_{y_1} - y_1\partial$$
$$x_{23}\partial_{x_{22}} + 2(x_{33} - x_{22})\partial_{x_{23}} - x_{23}\partial_{x_{33}} + x_{13}\partial_{x_{12}} - x_{12}\partial_{x_{13}} + y_3\partial_{y_2} - y_2\partial$$
$$r\partial_r - 2(x_{11}\partial_{x_{11}} + x_{12}\partial_{x_{12}} + x_{13}\partial_{x_{13}} + x_{22}\partial_{x_{22}} + x_{23}\partial_{x_{23}} + x_{33}\partial_{x_{33}})$$
$$-(y_1\partial_{y_1} + y_2\partial_{y_2} + y_3\partial_{y_3}) - 2.$$

Astronomical data

The astronomical data consist of the locations of 188 stars of magnitude brighter than or equal to 3.0.
Minimize

$$F(x, y, 1) \exp\left(-\sum_{1 \leq i \leq j \leq 3} S_{ij} x_{ij} - \sum_i S_i y_i\right)$$

on

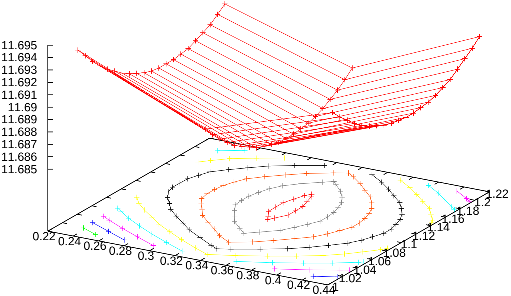
$$\begin{aligned} & (x_{11}, x_{12}, x_{13}, x_{22}, x_{23}, x_{33}, y_1, y_2, y_3) \\ \in \ & [-30, 10] \times [-30, 10] \times [-30, 10] \times [-30, 10] \times [-30, 20] \times [-30, - \\ & \times [-30, -0.01] \times [-30, -0.001] \times [-30, 10] \end{aligned}$$

where $(S_{11}, S_{12}, S_{13}, S_{22}, S_{23}, S_{33}, S_1, S_2, S_3) =$
$(0.3119, 0.0292, 0.0707, 0.3605, 0.0462, 0.3276, -0.0063, -0.0054, -0.076$

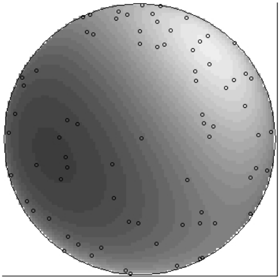
The result is that the minimum 11.68573121328159669 is taken at

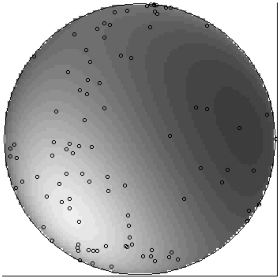
$$x = \begin{pmatrix} -0.161 & 0.3377/2 & 1.1104/2 \\ 0.3377/2 & 0.2538 & 0.6424/2 \\ 1.1104/2 & 0.6424/2 & -0.0928 \end{pmatrix},$$

$$y = (\underline{-0.019}, \underline{-0.0162}, -0.2286)$$



Estimated distribution





本格的な勉強のためのガイド

積分アルゴリズム, Pfaffian の導出, gradient の計算.

1. 高山信毅, D 加群の積分アルゴリズムと推定理論, 数学セミナー 2012.02, 41–46.
2. D.Cox, J.Little, D.O'Shea, Ideals, Varieties and Algorithms, Springer, 1992. 日本語訳あり. グレブナー基底の基礎 1,2 章.
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4. 道場, 第 6 章. 微分作用素環のグレブナー基底. 6.1–6.5.
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6. 道場, 6.5 HGD

Fisher-Bingham 分布の研究

1. Hiromasa Nakayama, Kenta Nishiyama, Masayuki Noro, Katsuyoshi Ohara, Tomonari Sei, Nobuki Takayama, Akimichi Takemura, Holonomic Gradient Descent and its Application to the Fisher-Bingham Integral, arxiv:1005.5273, Advances in Applied Mathematics 47 (2011), 639–658 HGD の方法論の提案も
2. T. Koyama, A Holonomic Ideal Annihilating the Fisher-Bingham Integral, http://arxiv.org/abs/1104.1411
3. T. Koyama, H. Nakayama, K. Nishiyama, N. Takayama, Holonomic Gradient Descent for the Fisher-Bingham Distribution on the *n*-dimensional Sphere, http://arxiv.org/abs/1201.3239
4. T. Koyama, H. Nakayama, K. Nishiyama, N. Takayama, The Holonomic Rank of the Fisher-Bingham System of Differential Equations, http://arxiv.org/abs/??

高次元で HGD でないもの.

1. K. Kume, A. T. A. Wood, Saddlepoint Approximations for the Bingham and Fisher-Bingham Normalising Constants, Biometrika **92** (2005), 465–476.

Fisher 分布 (HGD), Wishart 分布 (HGM)

1. Tomonari Sei, Hiroki Shibata, Akimichi Takemura, Katsuyoshi Ohara, Nobuki Takayama, Properties and applications of Fisher distribution on the rotation group, http://arxiv.org/abs/1110.0721
2. Hiroki Hashiguchi, Yasuhide Numata, Nobuki Takayama, Akimichi Takemura, Holonomic gradient method for the distribution function of the largest root of a Wishart matrix, http://arxiv.org/abs/1201.0472

HGD でないもの.

1. R.Butler, W.Ronald, A.T.A.Wood, Laplace approximations for hypergeometric functions with matrix argument, The Annals of Statistics (2002), 1155–1177.

2. Wishart 分布については, 基本問題のため, その他多数あり. 上記論文の Reference を参照.