

2008
1/12
9:39 -

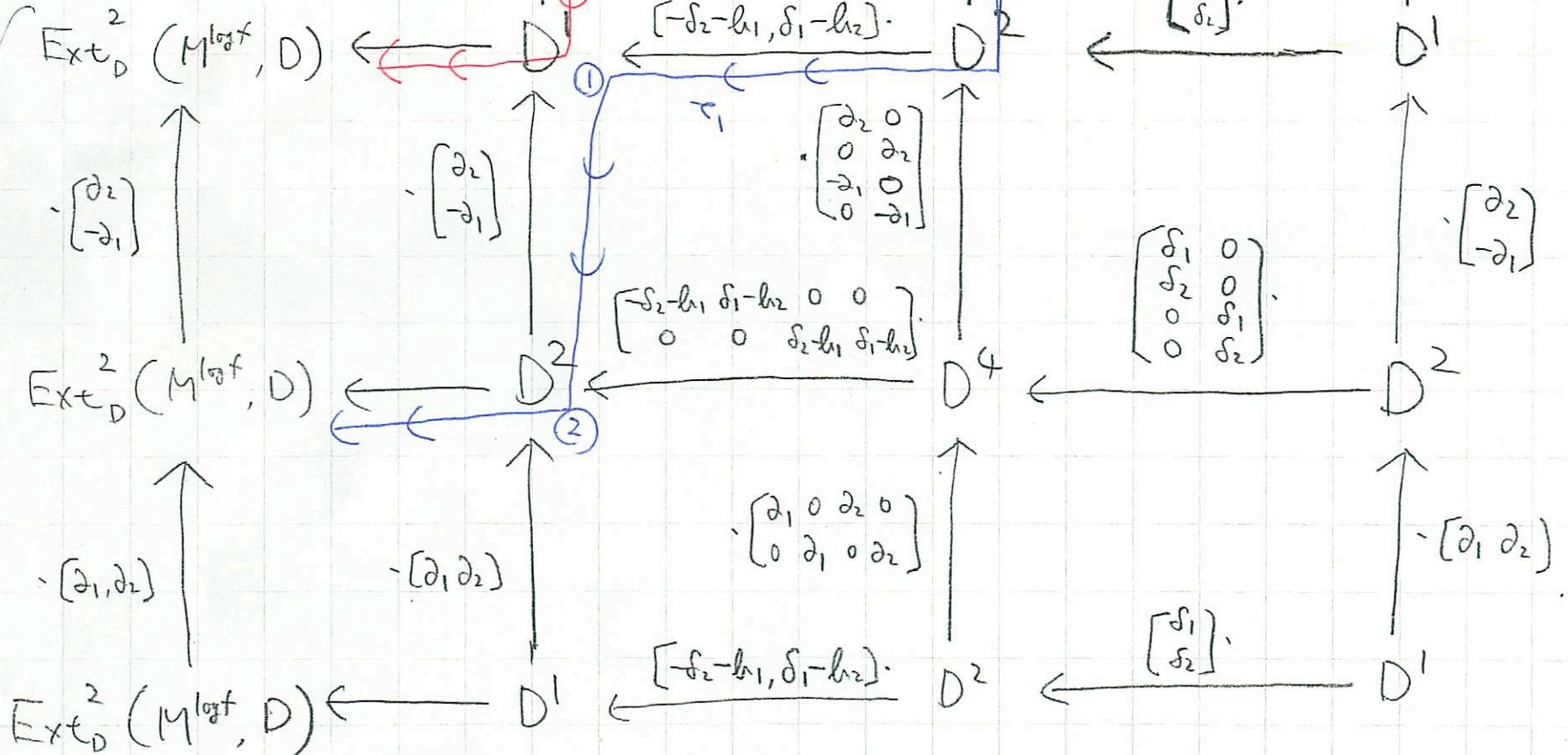
Ref. Tsai-Walther. p.603.

$$x_1 = x, x_2 = y.$$

- action $\rightarrow K(x_1, x_2)$ ①
- multiplication in D .

Derivation of explicit form of τ^i

right D -module.
↓



horizontal. D^m is regarded as a column vector.

Vertical " "

row " .

de Rham. for the right module.

$$D = K(x_1, x_2, \partial_1, \partial_2)$$

Red.

$$g \in K(x_1, x_2)$$

$$\begin{array}{c} \uparrow \\ \leftarrow D \\ g \end{array}$$

$$g \leftrightarrow g dx_1 dx_2$$

$$= g f \frac{dx_1 dx_2}{f}$$

$$= g w_1 \wedge w_2.$$

 τ_2

Solve

$$P_1 \partial_2 - P_2 \partial_1 = -h_1 \delta_2 + h_2 \delta_1$$

where $(P_1, P_2) \in D^2$ (2)
(unknown)

$$-h_1 \delta_2 + h_2 \delta_1$$

$$= -h_1 a_{21} \partial_1 - h_1 a_{22} \partial_2 + h_2 a_{11} \partial_1 + h_2 a_{12} \partial_2$$

$$= (-h_1 a_{21} + h_2 a_{11}) \partial_1 + (-h_1 a_{22} + h_2 a_{12}) \partial_2$$

Then, $P_1 = -h_1 a_{22} + h_2 a_{12}$
 $P_2 = h_1 a_{21} - h_2 a_{11}$

Blue:

$$(h_1, h_2) \in K(x_1, x_2)^2$$

$$(-\delta_2 - h_1) \cdot h_1 + (\delta_1 - h_2) \cdot h_2 = 0$$

∴

$$(-\delta_2 - h_1) \cdot h_1 + (\delta_1 - h_2) \cdot h_2$$

$$= -h_1 \delta_2 - \delta_2 \cdot h_1 - h_1 \delta_1$$

$$+ h_2 \cdot \delta_1 + \delta_1 \cdot h_2 - h_2 \delta_2$$

$$= -h_1 \delta_2 + h_2 \delta_1 \in D^1 \textcircled{1}$$

On the other hand,

$$fw_1 = a_{22} dx_1 - a_{21} dx_2$$

$$fw_2 = -a_{12} dx_1 + a_{11} dx_2$$

Then,

$$h_1 fw_1 + h_2 fw_2$$

$$= (h_1 a_{22} - h_2 a_{12}) dx_1$$

$$+ (-h_1 a_{21} + h_2 a_{11}) dx_2$$

$$= -P_1 dx_1 - P_2 dx_2$$

 τ_1

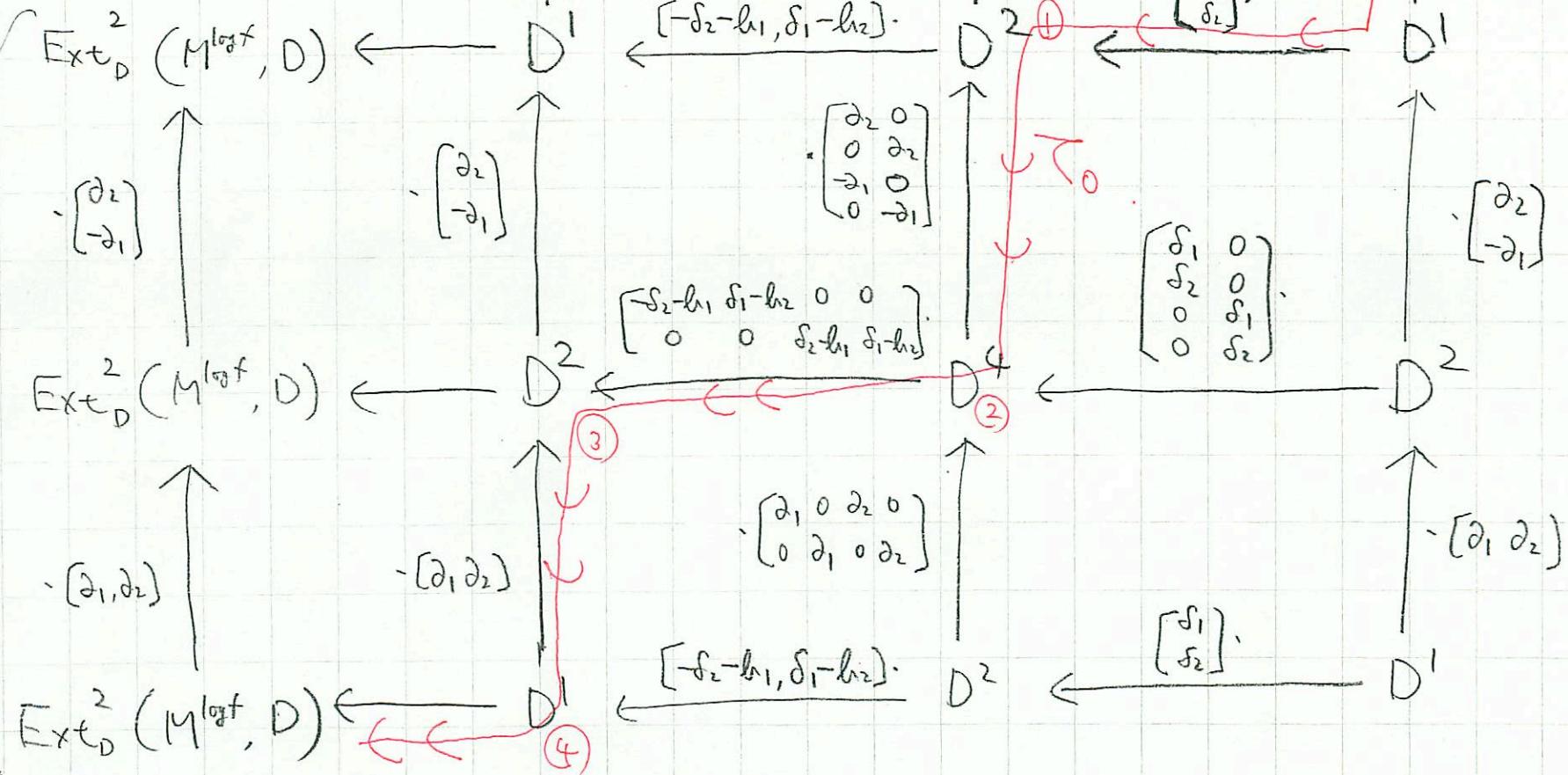
2008
1/9 全
9:39 -

Ref. Tsai-Walther. p.603.

$$x_1 = x, x_2 = y.$$

- action to $K(x_1, x_2)$ ③
 - multiplication in D .
- ← polynomial solution complex.

right D -module.
↓



horizontal. D^m is regarded as a column vector.

Vertical

"

row

"

$$D = K(x_1, x_2, \partial_1, \partial_2)$$

de Rham. for the right module

τ_0

$$g \in K[x_1, x_2]$$

$$\delta_1 \cdot g = 0, \quad \delta_2 \cdot g = 0$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \cdot g = g \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \in D^2 \quad \textcircled{1}$$

Solve

$$(f_1, f_2, f_3, f_4) \begin{pmatrix} \partial_2 & 0 \\ 0 & \partial_2 \\ \partial_1 & 0 \\ 0 & -\partial_1 \end{pmatrix} = g \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$$

$$f_1 = g a_{12} \quad f_2 = g a_{22}$$

$$f_3 = -g a_{11} \quad f_4 = -g a_{21}$$

$$(f_1, f_2, f_3, f_4) \in D^4 \quad \textcircled{2}$$

 \downarrow

$$\left((-\delta_2 - h_1) g a_{12} + (\delta_1 - h_2) g a_{22} \right. \\ \left. (\delta_2 + h_1) g a_{11} - (\delta_1 - h_2) g a_{21} \right) \in D^2 \quad \textcircled{3}$$

Since $\delta_1 \cdot g = \delta_2 \cdot g = 0$,
we may show that

$$f_2 = (-\delta_2 - h_1) a_{12} + (\delta_1 - h_2) a_{22}$$

$$f_2 = (\delta_2 + h_1) a_{11} - (\delta_1 - h_2) a_{21}$$

in order to prove that $\tau_0(g) = gf$.

page ⑤

$$\begin{aligned} & (-\delta_2 - h_1) a_{12} + (\delta_1 - h_2) a_{22} \\ &= -a_{12} \delta_2 - h_1 a_{12} + a_{22} \delta_1 - h_2 a_{22} \\ &\quad - \delta_2 \cdot a_{12} + \delta_1 \cdot a_{22} \\ &= -a_{12} a_{21} \cancel{a_1} - \cancel{a_1 a_{22} a_2} - h_1 a_{12} + a_{22} a_{11} \cancel{a_1} + \cancel{a_{11} a_{12} a_2} \\ &\quad - h_2 a_{22} \end{aligned}$$

$$- a_{21} a_{12,x} - a_{22} a_{12,y} + a_{11} a_{22,x} + a_{12} a_{22,y}$$

$$= f_1 \partial_1 - h_1 a_{12} - h_2 a_{22}$$

$$- a_{21} a_{12,x} - a_{22} a_{12,y} + a_{11} a_{22,x} + a_{12} a_{22,y}$$

$$= f_1 \partial_1$$

$$\nearrow \delta_1 \delta_2 - \delta_2 \delta_1 = h_1 \delta_1 + h_2 \delta_2 \text{ w.r.t. } \partial_2$$

coeff of

$$(\delta_2 + h_1) a_{11} - (\delta_1 - h_2) a_{21}$$

$$= a_{11} (\delta_2 + h_1) - a_{21} (\delta_1 - h_2)$$

$$+ \delta_2 \cdot a_{11} - \delta_1 \cdot a_{21}.$$

$$= \cancel{a_{11} a_{21} \delta_1} + \cancel{a_{11} a_{22} \delta_2} + h_1 a_{11} - \cancel{a_{21} a_{11} \delta_1} - \cancel{a_{21} a_{12} \delta_2} + h_2 a_{21}$$

$$+ a_{21} a_{11,x} + a_{22} a_{11,y} - a_{11} a_{21,x} - a_{12} a_{21,y}$$

$$= f \delta_2 + h_1 a_{11} + h_2 a_{21}$$

$$+ a_{21} a_{11,x} + a_{22} a_{11,y} - \cancel{a_{11} a_{21,x}} - \cancel{a_{12} a_{21,y}}$$



coeff of δ_1 of $\delta_1 \delta_2 - \delta_2 \delta_1 - (h_1 \delta_1 + h_2 \delta_2) = 0$.

$$= f \delta_2$$

//

$$\tau_0(g) = gf$$

Question.

$$\langle (-\delta_1 - h_1)^*, (\delta_1 - h_2)^* \rangle$$

$$= \widetilde{\text{Der}(\log f)} ?$$

Here * denotes the formal
adjoint.