

2007
7/23

```
import("glib3.rr");
Cfep_loaded=0$
glib_math_coordinate = 1;
glib_window(0,-2,2.0,2.0);
glib_line(0,0,2.0,0 | color=0xff0000);
X=0;
P=1;
Y=0;
Z=5;
H=0.1;
for (K = 0; K<20; K++) {
  print([X,P,Y,Z]);
  Xnew = X+H*P;
  Pnew = P;
  Ynew = Y+H*Z;
  Znew = Z-9.8*H;
  glib_line(X,Y,Xnew,Ynew);
  X = Xnew;
  P = Pnew;
  Y = Ynew;
  Z = Znew;
  glib_flush();
}
```

```
load("glib3.rr");
Cfep_loaded=0$

def lorentz() {
  glib_window(-25,-25,25,25);
  A=10; B=20; C=2.66;
  P1=0; P2 = 3; P3 = 0;
  Dt = 0.004; T = 0;
  while (T < 50) {
    Q1=P1+Dt*(-A*P1+A*P2);
    Q2=P2+Dt*(-P1*P3+B*P1-P2);
    Q3=P3+Dt*(P1*P2-C*P3);
    glib_putpixel(Q1,Q2);
    T=T+Dt;
    P1=Q1; P2=Q2; P3=Q3;
  }
}

lorentz()$

end$
```

7/24

B317.0200

```

/* x^2+y^2-4=0, x*y-1=0 を解く */
def newton2() {
  X=2.0; Y=0.0;
  for (I=1; I<=10; I++) {
    D=2*(X^2-Y^2);
    print([X,Y]);
    print([subst(x^2+y^2-4,x,X,y,Y),subst(x*y-1,x,X,y,Y)]);
    P = X-(X*(X^2+Y^2-4)-2*Y*(X*Y-1))/D;
    Q = Y-(-Y*(X^2+Y^2-4)+2*X*(X*Y-1))/D;
    X = P;
    Y = Q;
  }
}
end$

/* 実行結果.
asir

[1271] load("newton2.rr");
1
[1195] newton2();
[2,0]
[0,-1] 代入した時のずれ.

[2,0.5]
[0.25,0] 代入した時のずれ.

[1.93333,0.516667]
[0.00472222,-0.00111111] 代入した時のずれ.

[1.93185,0.517637]
[3.13381e-006,-1.43675e-006] 代入した時のずれ.

[1.93185,0.517638]
[2.25747e-012,-1.1271e-012] 代入した時のずれ.

[1.93185,0.517638]
[-4.996e-016,0] 代入した時のずれ.

省略.
*/

```

```

/* Ex: Heat_N=20; heat(0.001,30); CFL = 0.4 */
/* Ex: Heat_N=20; heat(0.003,30); CFL > 0.5 unstable */
import("glib3.rr")$
Glib_math_coordinate=1$
Cfep_loaded=0$

Heat_N=20$ /* Mesh size */

def heat(K,M) {
  extern Heat_N;
  H = 1.0/Heat_N; /* dx */
  print("dt=K=",0); print(K);
  print("CFL condition K/(H*H) (<= 0.5): ",0); print(K/(H*H));

  A = newvect(Heat_N+1);
  B = newvect(Heat_N+1);

  A[0] = 0; A[Heat_N] = 0;
  /* Setting initial condition. */
  for (Q=1; Q<Heat_N; Q++) {
    if (Q <= idiv(Heat_N,2)) {
      A[Q] = H*Q;
    } else {
      A[Q] = 1-H*Q;
    }
  }

  print("Initial vector: ",0); print(A);

  glib_window(0,0,Heat_N,1);
  glib_clear();
  for (P=1; P<=M; P++) {
    B[0] = 0; B[Heat_N]=0;
    for (Q=1; Q<Heat_N; Q++) {
      B[Q] = A[Q] + (K/(H*H))*(A[Q+1]-2*A[Q]+A[Q-1]);
    }
    print("Time=",0); print(P*K,0); print(":",0); print(B);
    /* code for DISPLAY */
    C = [ ];
    for (I=0; I<Heat_N+1; I++) {
      C = append(C,[I,B[I]]);
    }
    plot_dots(C);
    A = B;
  }
}

def plot_dots(C) {
  /* print(C); */
  N = length(C);
  for (I=0; I<N-1; I++) {
    P=C[I]; Q=C[I+1];
    glib_line(P[0],P[1],Q[0],Q[1] | color=0xff0000);
  }
}

heat(0.001,20);

```

```

Jul 25 2007 9:20                               peq.rr                               Page 1
/* $Id: peq.rr,v 1.1 2003/12/23 06:14:39 taka Exp $ */
/* x^3-x^2+x-8
F=x^3-x^2*t+x*t-8 = 0
df/dt = (dx/dt)*3*x^2 - (dx/dt)*2*x*t - x^2 + (dx/dt)*t + x = 0
dx/dt = (x^2-x)/(3*x^2-2*x*t+t)
[ 2.19928031473653232022 (-0.59964015736826616011+1.81052071783850331498*i) (-0.5996
4015736826616011-1.81052071783850331498*i) ]
*/
extern Cc $
Cc = 8 $
def homot() {
  Ans = [];
  /* The first solution */
  Xs = 2.0;
  Ans = append(Ans, [homot_aux(Xs)]);
  print("-----");

  /* The second solution */
  Xs = (-1.00000000000000000000+1.73205080756887729346*i);
  Ans = append(Ans, [homot_aux(Xs)]);
  print("-----");

  /* The third solution */
  Xs = (-1.00000000000000000000-1.73205080756887729346*i);
  Ans = append(Ans, [homot_aux(Xs)]);
  print("-----");
  return Ans;
}
def homot_aux(Xs) {
  X0 = Xs;
  H = 0.1; T = 0.0;
  while (1) {
    X1 = X0+H*((X0^2-X0)/(3*X0^2-2*X0*T+T));
    T += H;
    print("T=",0); print(T,0);
    print("old=",0); print(X1);
    X1=newton_aux(X1,T);
    print("new=",0); print(X1);
    E=T-1.0; if (E < 0.0) E = -E;
    if (E < 0.0001) break;
    X0 = X1;
  }
  return X1;
}
def newton_aux(A,T) {
  Epsilon = 0.0001;
  Q = A;
  P = Q+1.0;
  while (!( (Q-P > -Epsilon) &&
            (Q-P < Epsilon))) {
    P = Q;
    print(Q);
    Q = P-(P^3-P^2*T+P*T-Cc)/(3*P^2-2*P*T+P);
  }
  return(Q);
}

```

Risa/Asir (Win版)

7> f1u (file)
 ↓
 開く (open)
 2> peq.rr を D+ (load)
 homot(); [enter]
 実行開始
 (Start execution)

← cfep/Asir2.5.2に
 homot(); を追記

```

Jul 25 2007 9:36                               phc-pt3.txt                               Page 1
2;
9*(-2*u1+u2)+10*u1*(1-u1);
9*(u1-2*u2)+10*u2*(1-u2);
) ← input (x0)

THE SOLUTIONS :
3 2
=====
solution 1 :
t : 1.0000000000000000E+00  0.0000000000000000E+00
m : 1
the solution for t :
u1 : 1.0000000000000000E-01  -9.18354961579912E-41
u2 : 1.0000000000000000E-01  -9.18354961579912E-41
== err : 1.315E-17 = rco : 5.263E-02 = res : 2.776E-17 ==
solution 2 :
t : 1.0000000000000000E+00  0.0000000000000000E+00
m : 1
the solution for t :
u1 : -8.5000000000000000E-01  8.98610037780571E-01
u2 : -8.5000000000000000E-01  -8.98610037780571E-01
== err : 1.669E-16 = rco : 3.120E-01 = res : 4.441E-16 ==
solution 3 :
t : 1.0000000000000000E+00  0.0000000000000000E+00
m : 1
the solution for t :
u1 : -8.5000000000000000E-01  -8.98610037780571E-01
u2 : -8.5000000000000000E-01  8.98610037780571E-01
== err : 5.563E-16 = rco : 3.120E-01 = res : 6.661E-16 ==

```

output.

not
real
sol.

PHC pack.

phc -la phc-pt3.txt phc-pt3-out.txt [enter]

2007
7/26.

~~2007
7/26~~

```

import("gr")$
F=[x^2+y^2-4,x*y-1];
V=[x,y];
G=nd_gr(F,V,0,0);
print(["zero dim?", zero_dim(G,V,0)])$
M=dp_mbase(map(dp_ptod,G,V))$
print(M)$
print("Number of solutions is ",0)$
print(length(M))$
end$

```

← equations to solve.

← Output standard monomials
the

If you want to get a good grade, ...

Report problem. (レポート問題) due 8月10日.
to. takayama@math.kobe-u.ac.jp. (Do not send mail from private address)

1. Solve your favorite system of algebraic equations, with PHCpack or Risa/Asir.

or

2. Read the paper by B. Huber and B. Sturmfels, (hs.pdf) "A polyhedral method for solving sparse polynomial systems." 1995.

or
3. Try to read references cited in my lecture.

```

2;
9*(-2*u1+u2)+10*u1*(1-u1);
9*(u1-2*u2)+10*u2*(1-u2);

```

← input (x, y)

```

THE SOLUTIONS :
3 2
=====
solution 1 :
t : 1.000000000000000E+00  0.000000000000000E+00
m : 1
the solution for t :
u1 : 1.000000000000000E-01  -9.18354961579912E-41
u2 : 1.000000000000000E-01  -9.18354961579912E-41
== err : 1.315E-17 = rco : 5.263E-02 = res : 2.776E-17 ==
solution 2 :
t : 1.000000000000000E+00  0.000000000000000E+00
m : 1
the solution for t :
u1 : -8.500000000000000E-01  8.98610037780571E-01
u2 : -8.500000000000000E-01  -8.98610037780571E-01
== err : 1.669E-16 = rco : 3.120E-01 = res : 4.441E-16 ==
solution 3 :
t : 1.000000000000000E+00  0.000000000000000E+00
m : 1
the solution for t :
u1 : -8.500000000000000E-01  -8.98610037780571E-01
u2 : -8.500000000000000E-01  8.98610037780571E-01
== err : 5.563E-16 = rco : 3.120E-01 = res : 6.661E-16 ==

```

output.

not
real
sol.

PHC pack.

* phc -ln phc-pt3.txt phc-pt3-out.txt (c)

* $x^2 \Rightarrow x^{**2}$
 $x^3 \Rightarrow x^{**3}$

www.math.uic.edu/~jan/download.html

2007
7/25水.

$$\frac{U_{k+1} - 2U_k + U_{k-1}}{h^2} + a U_k (1 - u_k) = 0$$

$$U_0 = U_1 = 0.$$

$$0 \leq U_k \leq 1$$

$$h = \frac{1}{N}, \quad k=1, \dots, n-1$$

$$a = 10^3 \times 2. \quad \text{区々} \approx 10.$$

$$N=2, a \approx 10. \quad h = \frac{1}{2}$$

$$\frac{U_0 - 2U_1 + U_2}{\frac{1}{4}} + a U_1 (1 - u_1) = 0.$$

$$U_0 = U_2 = 0 \text{ として}$$

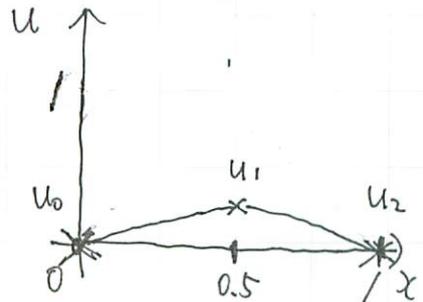
$$-8U_1 + a U_1 (1 - u_1) = 0$$

$$U_1 (-8 + a - a u_1) = 0$$

$$\therefore U_1 = 0, \quad \frac{a-8}{a} = 1 - \frac{8}{a}$$

$a = 10^3$.

$$U_1 = 1 - \frac{8}{10^3} = 0.2$$



non-zero. 近似解のグラフ。

(graph of approximate sol. for

$$N=3, a \approx 10. \quad h = \frac{1}{3}$$

$$u'' + a u(1-u) = 0$$

$$u(0) = u(1) = 0.$$

$$\left\{ \begin{array}{l} \frac{U_0 - 2U_1 + U_2}{\frac{1}{9}} + a U_1 (1 - u_1) = 0. \\ \frac{U_1 - 2U_2 + U_3}{\frac{1}{9}} + a U_2 (1 - u_2) = 0. \end{array} \right.$$

$$U_0 = U_3 = 0 \text{ として}$$

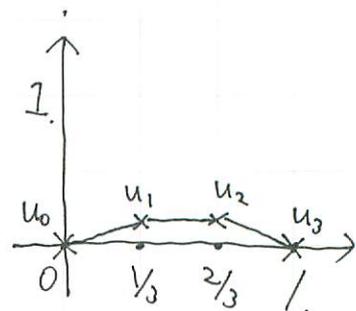
$$\left\{ \begin{array}{l} 9(-2u_1 + u_2) + a u_1 (1 - u_1) = 0. \\ 9(u_1 - 2u_2) + a u_2 (1 - u_2) = 0. \end{array} \right.$$

1st sol. $U_0 = U_1 = U_2 = U_3 = 0$

2nd sol. $U_0 = U_3 = 0.$

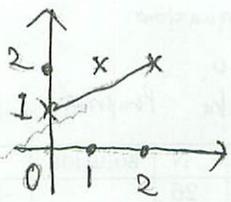
$$U_1 = U_2 \approx 0.1$$

← phc-pt3.txt.



2007
7/29
前日の補足

① $t^2 x^2 + t^2 x + t = 0$



Newton diagram

Cancel

$x = C_0 t^\sigma$

$t^2 C_0^2 t^{2\sigma} + t = 0$

$\therefore C_0^2 + 1 = 0, 2 + 2\sigma = 1$

$\therefore \sigma = -\frac{1}{2}, C_0 = \pm\sqrt{-1}$

$x = t^{-1/2} (C_0 + x_1(t))$ とおこ

①へ代入

$t^2 t^{-1} (C_0 + x_1(t))^2 + t^2 t^{-1/2} (C_0 + x_1(t)) + t = 0$

$t \cdot 2 C_0 x_1 + t x_1^2$

$+ t^{3/2} C_0 + t^{3/2} x_1 = 0$

x_1 をおこすと

$x_1 = C_1 t^{\sigma_1}$ とおこ

$2 C_0 t C_1 t^{\sigma_1} + C_0 t^{3/2} = 0$

$\therefore \sigma_1 + 1 = 3/2$

$\therefore \sigma_1 = 1/2$

$2 C_0 C_1 + C_0 = 0$

$\therefore C_1 = -1/2$

次に

$x_1(t) = t^{1/2} (C_1 + x_2(t))$

とおこ

x_2 に x_1 と同様の計算を

C_1 でおこ

x_1 までの計算を繰り返すと

$x(t) = t^{-1/2} (C_0 + t^{1/2} C_1 + t^{1/2} x_2)$

$= C_0 t^{-1/2} + C_1 t^0 + O(t^{1/2})$

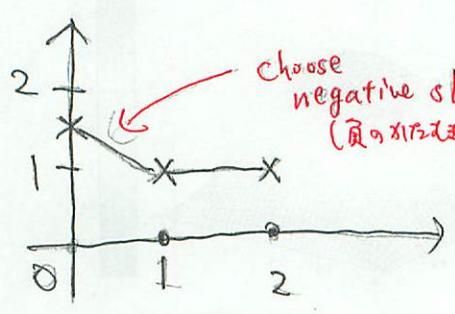
$\therefore C_0 = \pm\sqrt{-1}, C_1 = -1/2$

x_2 に x_1 と同様の計算を

$O(t^{1/2})$ の部分が $t \rightarrow 0$ とおこ

② $t x_1^2 + (2 C_0 t + t^{3/2}) x_1 + C_0 t^{3/2} = 0$

Newton Diagram をおこ



Reference: Walker, algebraic curves

p.97 ~ p.108.

Numerical solutions for
More complicated reaction-diffusion equations

than $u'' + \alpha u(1-u) = 0, u(0) = u(1) = 0.$

Ref. <http://www.math.kobe-u.ac.jp/~taka> Preprint. Mevisen, Kojima, Nie, Takayama

1. Homotopy (polyhedral)
2. Semi-definite relaxation
3. Grid refining.
4. Newton's method.

strategy	obj	N_N	N	solution	ϵ_{feas}	ϵ_{scaled}	m_e	t_C
init SPOP	F_5	10	26		-3e+1	-2e-1	2.09	203
mGrid 3a/b	F_M	10	51		-8e-1	-4e-2	-0.05	224
mGrid 3a/b	F_M	10	101		-3e-2	-4e-4	-0.02	383
mGrid 3a/b	F_M	10	201	4peak	-1e-8	-3e-11	-0.02	1082
init SPOP	F_1	10	26		-1e-1	-1e-3	-0.12	270
mGrid 3a/b	F_M	10	51		-1e-1	-4e-3	-0.08	348
mGrid 3a/b	F_M	10	101		-9e-12	-3e-16	-0.08	511
mGrid 3a/b	F_M	10	201	3peak	-5e-9	-2e-11	-0.07	1192

Table 4: Results for grid-refining strategy 3a/3b

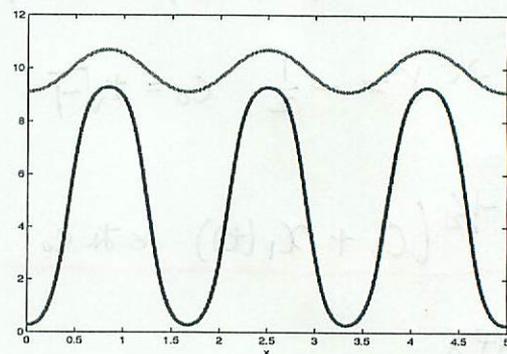
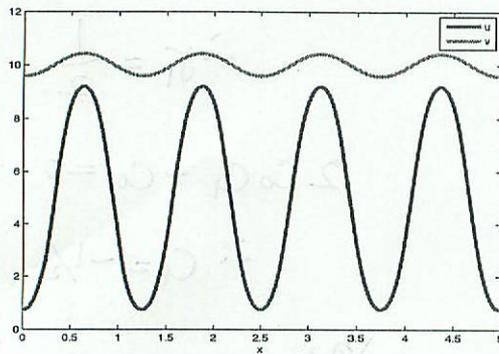


Figure 6: Stable solutions for grid-refining strategy 3a/b

We take the objective function as $F(u) = \sum_{i,j} u_{i,j}$ and choose $\omega = 2$ and $N_x = N_y = 6$ for the initial SparsePOP and obtain a highly accurate stable solution on a 41×41 -grid. The results are documented in Table 7 and pictured in Figure 9.

N_x	6	11	11	21	21	41	41
N_y	6	6	11	11	21	21	41
ϵ_{feas}	-6.4e-10	-6.2e-11	-1.2e-10	-8.3e-8	-8.9e-8	-3.7e-10	-3.8e-10
m_e	-5.61	-5.18	-4.74	-4.63	-4.51	-4.48	-4.46

Table 7: grid-refining strategy 3b for problem (4.5) in case $\lambda = 22$

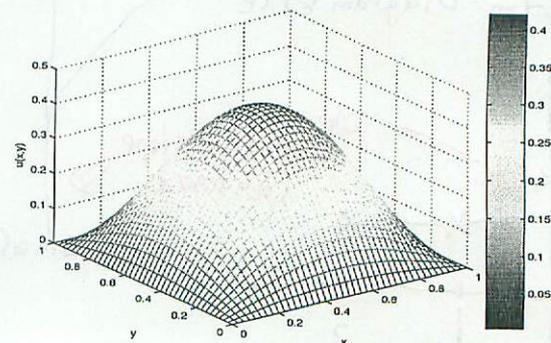
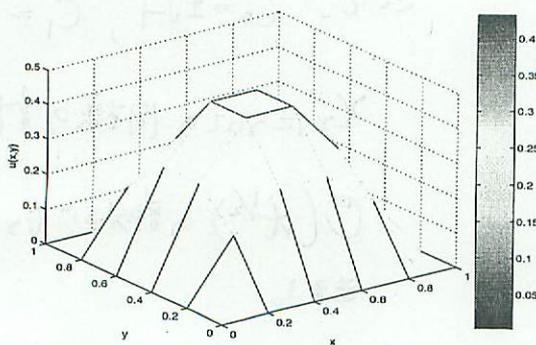


Figure 9: Solution for (4.5) if $\lambda = 22$, $(N_x, N_y) = (6, 6)$ and $(N_x, N_y) = (41, 41)$