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Small Data Global Existence of Solutions for Dissipative Wave Equations in an Exterior Domain

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Abstract

First we shall derive the basic decay estimates of the total energy and L^2-norm of a solution to the mixed problem for the linear dissipative wave equation in an exterior domain with the initial data satisfying some further restrictions as $|x| \to +\infty$. That decay estimates are faster than the usual one. Second we shall apply the decay estimates above to the exterior mixed problem of the semilinear dissipative wave equation in an exterior domain and we shall derive the small data global existence property to that problem with the power satisfying $1 + 4/(N+2) < p \leq N/(N-2)(N = 3, 4, 5)$ on the nonlinear term $|u|^p$.

1 Introduction

Let $\Omega \subset R^N(N \geq 3)$ be an exterior domain with compact smooth boundary $\partial\Omega$. Without loss of generality we may assume $0 \notin \bar{\Omega}$. In this paper we are concerned with the initial-boundary value problem for the linear dissipative wave equation:

$$v_{tt}(t,x) - \Delta v(t,x) + v_t(t,x) = 0, \quad (t,x) \in (0,\infty) \times \Omega, \tag{1.1}$$

$$v(0,x) = v_0(x), \quad v_t(0,x) = v_1(x), \quad x \in \Omega, \tag{1.2}$$

$$v|_{\partial\Omega} = 0, \quad t \in (0,\infty), \tag{1.3}$$

and the semilinear dissipative wave equation:

$$u_{tt}(t,x) - \Delta u(t,x) + u_t(t,x) = |u(t,x)|^p, \quad (t,x) \in (0,\infty) \times \Omega, \tag{1.4}$$

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \Omega, \tag{1.5}$$

$$u|_{\partial\Omega} = 0, \quad t \in (0,\infty). \tag{1.6}$$

Throughout this paper, $\|\cdot\|_q$ and $\|\cdot\|_{H^1}$ mean the usual $L^q(\Omega)$-norm and $H_0^1(\Omega)$-norm, respectively, and in particular, we set $\|\cdot\| = \|\cdot\|_2$ for simplicity. Furthermore, we adopt

$$(f,g) = \int_\Omega f(x)g(x)dx$$

as the usual $L^2(\Omega)$-inner product. The total energy $E_v(t)$ to the equation (1.1) and (1.4) is defined by

$$E_v(t) = \frac{1}{2}\|v_t(t,\cdot)\|^2 + \frac{1}{2}\|\nabla v(t,\cdot)\|^2.$$

The first purpose of this job is to derive certain decay estimates for the total energy $E_v(t)$ and the L^2-norm of a solution $v(t,x)$ to the linear problem (1.1)-(1.3) faster than the usual one through the (modified) time integral method developed in Ikehata-Matsuyama [3]. In that occasion, we do assume some further restrictions on the initial data as $|x| \to +\infty$. On the contrary, Ikehata-Matsuyama [3] and Saeki-Ikehata [11] adopted another weight condition on the initial data. For the exterior mixed problem, these restrictions on the initial data seem to be new (for conditions on the initial data with the compact support, see Dan-Shibata [1]). For these restrictions as $|x| \to +\infty$ on the initial data to the "Cauchy problem" of the equation (1.1), there are lots of related results and we refer the reader to Kawashima-Nakao-Ono [6], Matsumura [8] and the references thererein.

The second purpose of this paper is to determine the exponent p of the semilinear exterior problem (1.4)-(1.6) for which the small data global existence property holds. Very recently, in Ikehata-Miyaoka-Nakatake [4] and Todorova-Yordanov [12] they have derived such a critical (Fujita type) exponent $p_c = 1 + 2/N$ to the Cauchy problem of (1.4) in the framework of $L^1 \times L^1$ assumption on the initial data and of the initial data with compact support, respectively (for another type of critical exponents like $p_c = 1 + 2m/N$ for the Cauchy problem of (1.4) with $L^m \times L^m$ assumption on the initial data, see also Ikehata-Ohta [5]). These works are fully based on the decay estimates for the linear equations due to Matsumura [8] and Kawashima-Nakao-Ono [6]. Thus, it seems to be difficult to apply those decay estimates for the linear equations due to [6] and [8] to the present exterior mixed problem (1.4)-(1.6). On the other hand, in the framework of the compactly supported initial data Ikehata [2] has already constructed a small global solution to the exterior problem (1.4)-(1.6) with the power $1 + 6/(N+2) < p \leq N/(N-2)(N=3)$ or $1 + 6/(N+2) < p < +\infty(N=2)$. His result is based on the decay estimates for the linear equations which are developed in [3] and [11]. By using decay estimates for the linear equations developed in the former part instead of those developed in [3] and [11], we can exclude the compactness of the support on the initial data as in [2] to the problem (1.4)-(1.6) with further relaxed exponent, and we can also treat the higher dimensional case $N = 4, 5$ (for another exponent $p_c = 1 + 4/N$, see Nakao-Ono [10]).

In the following, we set

$$I_{0,u} = \|u_0\|_{H^1} + \|u_1\| + \|u_0 + u_1\|_{2N/(N+2)}.$$

Then our first result reads as follows.

Theorem 1.1 *Let* $N \geq 3$. *For each* $[v_0, v_1] \in (H_0^1(\Omega) \cap L^{2N/(N+2)}(\Omega)) \times (L^2(\Omega) \cap L^{2N/(N+2)}(\Omega))$*, the weak solution* $v \in C([0,+\infty); H_0^1(\Omega)) \cap C^1([0,+\infty); L^2(\Omega))$ *to the linear problem* (1.1)-(1.3) *satisfies the decay estimates:*

$$\|v(t,\cdot)\|^2 \leq C I_{0,v}^2 (1+t)^{-1},$$

$$\|v_t(t,\cdot)\|^2 + \|\nabla v(t,\cdot)\|^2 \leq C I_{0,v}^2 (1+t)^{-2} \tag{1.7}$$

with some generous constant $C > 0$.

Next we shall make some assumptions before treating the semilinear problem (1.4)-(1.6).

$$\mathbf{1}+\frac{\mathbf{4}}{\mathbf{N+2}}<\mathbf{p}\leq\frac{\mathbf{N}}{\mathbf{N-2}}, \tag{1.8}$$

Now based on these decay estimates for the linear equations as in Theorem 1.1 our second result to the semilinear problem reads as follows.

Theorem 1.2 *Let $N = 3, 4, 5$. Under the assumption* **(1.8)**, *there exists a real number $\delta > 0$ such that if the initial data $[u_0, u_1] \in (H_0^1(\Omega) \cap L^{2N/(N+2)}(\Omega)) \times (L^2(\Omega) \cap L^{2N/(N+2)}(\Omega))$further satisfies $I_{0,u} < \delta$, the problem* (1.4)-(1.6) *has a unique global solution $u \in C([0, +\infty); H_0^1(\Omega)) \cap C^1([0, +\infty); L^2(\Omega))$ satisfying*

$$\|u(t,\cdot)\|^2 \leq CI_{0,u}^2(1+t)^{-1},$$

$$\|u_t(t,\cdot)\|^2 + \|\nabla u(t,\cdot)\|^2 \leq CI_{0,u}^2(1+t)^{-2}$$

with some generous constant $C > 0$.

Remark 1.1 In the case when $N = 3$, Theorem 1.2 completely contains the result in Ikehata [2]. Furthermore, (1.8) is more relaxed condition than that in [2]. For $N = 2$, this is completely open. Note that in the case when $N = 2$, formally we get $1 + 4/(N + 2) = 1 + 2/N$. On the other hand, the results in Theorems 1.1 and 1.2 seem sharp if we take $m = 2N/(N+2)$ in Ikehata-Ohta [5, Theorem 1.2]. Finally, for the nonexistence of global solution to the equation (1.4) in $\mathbf{R}^N$ with power p less than the critical exponent we refer to Li-Zhou [7], Todorova-Yordanov [12] and Ikehata-Ohta [5].

2 Proof of Theorem 1.1

In this section we shall prove Theorem 1.1. Our discussion shall be based on the following well-posedness result to the mixed problem (1.1)-(1.3). This is rather standard.

Proposition 2.1 *Let $N \geq 2$. For each $[v_0, v_1] \in H_0^1(\Omega) \times L^2(\Omega)$, there exists a unique weak solution $v \in C([0, +\infty); H_0^1(\Omega)) \cap C^1([0, +\infty); L^2(\Omega))$ to the linear problem* (1.1)-(1.3) *satisfying*

$$E_v(t) + \int_0^t \|v_t(s,\cdot)\|^2 ds = E_v(0),$$

$$\frac{d}{dt}(v_t(t,\cdot), v(t,\cdot)) + \|\nabla v(t,\cdot)\|^2 + (v_t(t,\cdot), v(t,\cdot)) = \|v_t(t,\cdot)\|^2.$$

We use the following fundamental Sobolev inequality.

Lemma 2.1 *Let $N \geq 3$. For each $u \in H_0^1(\Omega)$ it holds that*

$$\|u\|_{2N/(N-2)} \leq C^*\|\nabla u\|$$

with some constant $C^* > 0$.

First we shall derive the L^2-decay estimate. The following lemma is known more or less and is already announced (at least) in Saeki-Ikehata [11].

Lemma 2.2 *Let $N \geq 2$. If $[v_0, v_1] \in H_0^1(\Omega) \times L^2(\Omega)$, the weak solution $v \in C([0, +\infty); H_0^1(\Omega)) \cap C^1([0, +\infty); L^2(\Omega))$ to the linear problem (1.1)-(1.3) satisfies*

$$(1+t)\|v(t,\cdot)\|^2 \leq C(\|v_0\|_{H^1} + \|v_1\|)^2 + C\int_0^t \|v(s,\cdot)\|^2 ds$$

with some constant $C > 0$ which is independent of the initial data.

The next lemma is crucial in our argument and this part plays an essential role in deriving Theorem 1.1.

Lemma 2.3 *Under the same assumptions as in Theorem 1.1, it holds that*

$$\|v(t,\cdot)\|^2 + \int_0^t \|v(s,\cdot)\|^2 ds \leq C\{\|v_0\| + \|v_0 + v_1\|_{2N/(N+2)}\}^2$$

with some constant $C > 0$.

Proof. The proof will be done along the same line as in Ikehata-Matsuyama [3] except for using Lemma 2.1 instead of the so called Hardy inequality. Indeed, set

$$w(t,x) = \int_0^t v(s,x)ds.$$

Then $w \in C^1([0, +\infty); H_0^1(\Omega)) \cap C^2([0, +\infty); L^2(\Omega))$ satisfies

$$w_{tt}(t,x) - \Delta w(t,x) + w_t(t,x) = v_0 + v_1, \quad (t,x) \in (0,\infty) \times \Omega, \tag{2.1}$$

$$w(0,x) = 0, \quad w_t(0,x) = v_0(x), \quad x \in \Omega, \tag{2.2}$$

$$w|_{\partial\Omega} = 0, \quad t \in (0,\infty), \tag{2.3}$$

and

$$\frac{1}{2}\|w_t(t,\cdot)\|^2 + \frac{1}{2}\|\nabla w(t,\cdot)\|^2 + \int_0^t \|w_t(s,\cdot)\|^2 ds$$

$$= \frac{1}{2}\|v_0\|^2 + \int_0^t (v_0 + v_1, w_t(s,\cdot))ds. \tag{2.4}$$

Now because of (2.2) we have

$$\int_0^t (v_0 + v_1, w_t(s,\cdot))ds = (w(t,\cdot), v_0 + v_1),$$

and from the Hölder inequality we see

$$(w(t,\cdot), v_0 + v_1) \leq \|w(t,\cdot)\|_{2N/(N-2)}\|v_0 + v_1\|_{2N/(N+2)}.$$

So, by using Lemma 2.1 and (2.4) one has

$$\frac{1}{2}\|w_t(t,\cdot)\|^2 + \frac{1}{2}\|\nabla w(t,\cdot)\|^2 + \int_0^t \|w_t(s,\cdot)\|^2 ds$$

$$\leq \frac{1}{2}\|v_0\|^2 + \frac{C_1}{\epsilon}\|v_0 + v_1\|^2_{2N/(N+2)} + \frac{C_2\epsilon}{2}\|\nabla w(t,\cdot)\|^2$$

with some constants $C_i > 0 (i = 1, 2)$. Because of $w_t = v$, this inequality implies

$$\frac{1}{2}\|v(t,\cdot)\|^2 + \frac{1}{2}(1 - C_2\epsilon)\|\nabla w(t,\cdot)\|^2 + \int_0^t \|v(s,\cdot)\|^2 ds$$

$$\leq \frac{1}{2}\|v_0\|^2 + \frac{C_1}{\epsilon}\|v_0 + v_1\|^2_{2N/(N+2)}$$

for any $\epsilon > 0$. Taking $\epsilon > 0$ so small, we have arrived at the desired estimate. ∎

Therefore, L^2-decay estimate in Theorem 1.1 is a direct consequence of Lemmas 2.2 and 2.3. Once we have obtained the L^2-decay estimate in Theorem 1.1, the energy estimate (1.7) is calculated along the same way as in Ikehata [2] and so, we shall omit its proof.

3 Proof of Theorem 1.2

In this section we will prove Theorem 1.2. We proceed our argument based on the following local well-posedness.

Proposition 3.1 *Let $N \geq 3$ and $1 < p \leq N/(N-2)$. Then for each $[u_0, u_1] \in H^1_0(\Omega) \times L^2(\Omega)$, there exists a maximal existence time $T_m > 0$ such that the problem (1.4)-(1.6) has a unique solution $u \in C([0, T_m); H^1_0(\Omega)) \cap C^1([0, T_m); L^2(\Omega))$. Furthermore, if $T_m < +\infty$, then it holds that*

$$\lim_{t \uparrow T_m} [\|u_t(t,\cdot)\| + \|u(t,\cdot)\|_{H^1}] = +\infty.$$

First we shall prepare several facts which come from Theorem 1.1 concerning the linear problem:

$$v_{tt}(t,x) - \Delta v(t,x) + v_t(t,x) = 0, \quad (t,x) \in (0,\infty) \times \Omega, \tag{3.1}$$

$$v(0,x) = v_0(x), \quad v_t(0,x) = v_1(x), \quad x \in \Omega, \tag{3.2}$$

$$v|_{\partial\Omega} = 0, \quad t \in (0,\infty). \tag{3.3}$$

Define a semigroup $S(t) : H^1_0(\Omega) \times L^2(\Omega) \to H^1_0(\Omega) \times L^2(\Omega)$ by

$$S(t) : \begin{bmatrix} v_0 \\ v_1 \end{bmatrix} \mapsto \begin{bmatrix} v(t,\cdot) \\ v_t(t,\cdot) \end{bmatrix},$$

where $v(t,\cdot) \in C([0,+\infty); H^1_0(\Omega)) \cap C^1([0,+\infty); L^2(\Omega))$ is a unique solution to the "linear" problem (3.1)-(3.3). The following two lemmas are direct consequences of Theorem 1.1. In the following context, $C > 0$ denotes the various generous constant.

Lemma 3.1 *Let $N \geq 3$. If $[v_0, v_1] \in H^1_0(\Omega) \times L^2(\Omega)$ further satisfies $\|u_0 + u_1\|_{2N/(N+2)} < +\infty$, then it holds that*

$$\|v(t,\cdot)\| \leq C I_{0,u}(1+t)^{-1/2}.$$

Set

$$\|\begin{bmatrix} u \\ v \end{bmatrix}\|_E = \|v\| + \|\nabla u\|.$$

Lemma 3.2 *Let $N \geq 3$. If $[u_0, u_1] \in H_0^1(\Omega) \times L^2(\Omega)$ further satisfies $\|u_0 + u_1\|_{2N/(N+2)} < +\infty$, then it holds that*

$$\|S(t)\begin{bmatrix} v_0 \\ v_1 \end{bmatrix}\|_E \leq C I_{0,u}(1+t)^{-1}.$$

In order to control the nonlinear term, we shall prepare the well-known Gagliardo-Nirenberg inequality.

Lemma 3.3 *Let $1 \leq r < q \leq 2N/(N-2)(N \geq 3)$, $2 \leq q$. Then, if $u \in H_0^1(\Omega)$, we have*

$$\|u\|_q \leq M^{1/p}\|u\|_r^{1-\theta}\|\nabla u\|^{\theta},$$

where $M > 0$ is a constant independent of u,

$$\theta = (1/r - 1/q)(1/N - 1/2 + 1/r)^{-1} \in (0, 1],$$

and p is a positive real number satisfying (1.8).

Futhermore, we shall prepare the following well-known inequalities. For the proof, see [2].

Lemma 3.4 *If $\beta > 1$, then there exists a constant $C_\beta > 0$ depending only on β such that*

$$(1) \quad \int_0^t (1+t-s)^{-\frac{1}{2}}(1+s)^{-\beta} ds \leq C_\beta (1+t)^{-\frac{1}{2}},$$

$$(2) \quad \int_0^t (1+t-s)^{-1}(1+s)^{-\beta} ds \leq C_\beta (1+t)^{-1}$$

for all $t \geq 0$.

Now based on these decay estimates for the linear problem (3.1)-(3.3) we shall derive the decay property of a nonlinear problem (1.4)-(1.6). By a standard semigroup theory, the nonlinear problem (1.4)-(1.6) is rewritten as:

$$U(t) = S(t)U_0 + \int_0^t S(t-s)F(s)ds, \tag{3.4}$$

where $U(t) = \begin{bmatrix} u(t,\cdot) \\ u_t(t,\cdot) \end{bmatrix}$, $U_0 = \begin{bmatrix} u_0 \\ u_1 \end{bmatrix}$, $F(s) = \begin{bmatrix} 0 \\ f(u(s,\cdot)) \end{bmatrix}$ with $f(u)(x) = |u(x)|^p$.

We proceed our argument based on the Nakao method [9]. In order to show the global existence, it suffices to obtain the a priori estimates for $E_u(t)$ and $\|u(t,\cdot)\|$ in the interval of existence $[0, T_m)$. For simplicity, we set $I_0 = I_{0,u}$. As a result of Lemmas 3.1 and 3.2, first one has

Lemma 3.5 *Under the assumptions as in Theorem 1.2, we have*

$$\|S(t)U_0\|_E \leq C I_0 (1+t)^{-1}$$

on $[0, T_m)$.

Furthermore, if

$$I(s) = \|f(u(s,\cdot))\| + \|f(u(s,\cdot))\|_{2N/(N+2)} < +\infty$$

for each $s \in [0,t]$ with $t \in [0,T_m)$, then from Lemma 3.2 we have

$$\|S(t-s)F(s)\|_E \le CI(s)(1+t-s)^{-1}. \tag{3.5}$$

Thus from (3.4) one can estimate $U(t)$ as follows:

$$\|U(t)\|_E \le CI_0(1+t)^{-1} + C\int_0^t (1+t-s)^{-1} I(s)ds. \tag{3.6}$$

Take $K > 0$ so large and choose $T \in (0,T_m)$ so small such as

$$(1+t)\|U(t)\|_E \le KI_0 \quad on \quad [0,T), \tag{3.7}$$

$$(1+t)^{1/2}\|u(t)\| \le KI_0 \quad on \quad [0,T). \tag{3.8}$$

Then because of Lemma 3.3 we can estimate such as

$$\|f(u(s,\cdot))\|_{2N/(N+2)} = \|u(s,\cdot)\|^p_{2pN/(N+2)} \le M\|u(s,\cdot)\|^{p(1-\theta_1)}\|\nabla u(s,\cdot)\|^{p\theta_1}$$

with $\theta_1 = (Np-N-2)/2p \in (0,1]$. Similarly one has

$$\|f(u(s,\cdot))\| \le M\|u(s,\cdot)\|^{p(1-\theta_2)}\|\nabla u(s,\cdot)\|^{p\theta_2}$$

with $\theta_2 = N(p-1)/2p \in (0,1]$. Therefore, as long as (3.7)-(3.8) hold one gets

$$\begin{aligned}\|f(u(s,\cdot))\|_{2N/(N+2)} &\le M\{KI_0(1+s)^{-1/2}\}^{p(1-\theta_1)}\{KI_0(1+s)^{-1}\}^{p\theta_1} \\ &= MK^pI_0^p(1+s)^{-p(1+\theta_1)/2}\end{aligned}$$

and

$$\begin{aligned}\|f(u(s,\cdot))\| &\le M\{KI_0(1+s)^{-1/2}\}^{p(1-\theta_2)}\{KI_0(1+s)^{-1}\}^{p\theta_2} \\ &= MK^pI_0^p(1+s)^{-p(1+\theta_2)/2}.\end{aligned}$$

Setting $\gamma_i = p(1+\theta_i)/2 (i=1,2)$, we have

Lemma 3.6 *As long as* (3.7)-(3.8) *hold on* $[0,T)$ *we have*

$$\|f(u(t,\cdot))\|_{2N/(N+2)} \le MK^pI_0^p(1+t)^{-\gamma_1},$$

$$\|f(u(t,\cdot))\| \le MK^pI_0^p(1+t)^{-\gamma_2}.$$

So, by applying Lemma 3.6 to (3.6) we see that

$$\|U(t)\|_E \le CI_0(1+t)^{-1} + CMK^pI_0^p\int_0^t (1+t-s)^{-1}\{(1+s)^{-\gamma_1} + (1+s)^{-\gamma_2}\}ds.$$

Note that $\gamma_1 < \gamma_2$ in the present case. Thus, we have

$$\|U(t)\|_E \le CI_0(1+t)^{-1} + CMK^pI_0^p\int_0^t (1+t-s)^{-1}(1+s)^{-\gamma_1}ds$$

with some generous constant $C > 0$. On the other hand, we see that $\gamma_1 > 1$ because of the assumption **(1.8)**, so that from Lemma 3.4 it follows that

$$\|U(t)\|_E \le CI_0(1+t)^{-1} + CMK^pI_0^p(1+t)^{-1}.$$

Setting

$$Q_0(I_0, K) = C + CMK^pI_0^{p-1},$$

we get the following lemma.

Lemma 3.7 *As long as* (3.7)-(3.8) *hold on* $[0, T)$ *we get*

$$\|U(t)\|_E \le I_0Q_0(I_0, K)(1+t)^{-1}.$$

Next let us derive the L^2-estimates for the local solution $u(t, x)$ to the problem (1.4)-(1.6). Indeed, we have from (3.4) and Lemma 3.1 that

$$\|u(t,\cdot)\| \le CI_0(1+t)^{-1/2} + C\int_0^t (1+t-s)^{-1/2}I(s)ds.$$

Therefore, it follows from Lemma 3.6 that

$$\|u(t,\cdot)\| \le CI_0(1+t)^{-1/2}$$

$$+CMK^pI_0^p\int_0^t (1+t-s)^{-1/2}\{(1+s)^{-\gamma_1} + (1+s)^{-\gamma_2}\}ds$$

with some generous constant $C > 0$. Since one has $\gamma_1 < \gamma_2$ and $\gamma_1 > 1$ again because of **(1.8)**, this together with Lemma 3.4 implies

$$\|u(t,\cdot)\| \le CI_0(1+t)^{-1/2} + CMK^pI_0^p(1+t)^{-1/2}.$$

Thus we have

Lemma 3.8 *As long as* (3.7)-(3.8) *hold on* $[0, T)$ *it follows that*

$$\|u(t,\cdot)\| \le I_0Q_0(I_0, K)(1+t)^{-1/2}.$$

Take $K > C$ so large and take I_0 so small such as

$$CMK^pI_0^{p-1} < K - C. \tag{3.9}$$

For such $K > 0$ and I_0 we have

$$Q_0(I_0, K) < K.$$

Therefore, by combining this with Lemmas 3.7 and 3.8 we see that

$$\|U(t)\|_E < KI_0(1+t)^{-1}, \tag{3.10}$$

$$\|u(t,\cdot)\| < KI_0(1+t)^{-1/2} \tag{3.11}$$

on $[0, T)$. (3.7)-(3.8) and (3.10)-(3.11) show that under the assumption (3.9), the local solution $u(t, \cdot)$ exists globally in time and these estimates hold in fact for all $t \ge 0$. Taking $\delta = (\frac{K-C}{CMK^p})^{1/(p-1)}$, the proof of Theorem 1.2 is now finished.

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