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FORCED VIBRATIONS OF ABSTRACT WAVE EQUATIONS

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Abstract. Existence results of periodic solutions of certain abstract nonlinear wave equations are given when eigenvalues of linear parts of those equations are incommensurable to the time period of forcing terms.

1. Introduction

In this paper, we investigate the existence of periodic solutions for certain abstract wave equations. We are motivated by the papers of K. Ben-Naoum and J. Mawhin [1], and P.J. McKenna [8], where existence results of periodic solutions are proved for one-dimensional wave equations when the ratio between the space length and the period was irrational. Related equations are also studied by M. Yamaguchi in [10]. We proceeded in this direction in the paper [4]. We studied the equation

$$u_{tt} + Au = \varepsilon f(u,t)\,, \tag{1.1}$$

where A is a self-adjoint, unbounded linear operator with eigenvalues $\lambda_1 \leq \lambda_2 \leq \cdots \to \infty$, ε is a small parameter and f is T-periodic in $t \in \mathbb{R}$. By a T-periodic solution of (1.1) we mean a weak solution specified below. The following results are proved in [4] under additional assumptions on A, f.

Theorem 1.1. ([4]) *Assume there exists a constant $c > 0$ such that*

$$\left|\alpha^2 - \frac{m^2}{\lambda_i}\right| \geq \frac{c}{\lambda_i} \qquad \forall\, m \in \mathbb{N}, \quad \forall\, \lambda_i > 0\,, \tag{1.2}$$

where $\alpha = \frac{T}{2\pi}$. Then (1.1) has a weak T-periodic solution for any ε small.

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Theorem 1.2. ([4]) *Assume*

$$\sum_{\lambda_i>0} \frac{1}{\sqrt{\lambda_i}} < \infty \,.$$

Then the Lebesque measure of the set of all positive α not satisfying (1.2) *is zero.*

We also studied the case when $0 < \dim\ker A < \infty$. Finally we considered the example

$$\begin{aligned} &u_{tt} - u_{xx} - n^2 u = \varepsilon f(u,t) \\ &u(t+T,\cdot) = u(t,\cdot) \quad \forall\, t \in S^T \\ &u(t,0) = u(t,\pi) = 0 \quad \forall\, t \in S^T \,, \end{aligned} \tag{1.3}$$

where $f\colon \mathbb{R}\times S^T \to \mathbb{R}$ is C^1-smooth and globally Lipschitz in u, $n \in \mathbb{N}$. Here $S^T = \mathbb{R}/[0,T]$ is the circle. The following result is proved in [4].

Theorem 1.3. ([4]) *The equation* (1.3) *has a weak T-periodic solution, provided that it holds*

$$\inf_{i,m\in\mathbb{N},i>n} |i^2 - n^2 - \omega^2 m^2| > 0 \,, \tag{1.4}$$

where $\omega = 1/\alpha$, and there is a $z \in \mathbb{R}$ such that

$$\begin{aligned} &\int_0^T\!\!\int_0^\pi f(z.\sin nx, t)\sin nx\,dx\,dt = 0 \\ &\int_0^T\!\!\int_0^\pi \frac{\partial f}{\partial u}(z.\sin nx, t)\sin^2 nx\,dx\,dt \neq 0 \,. \end{aligned}$$

The purpose of this paper is two-fold. We firstly release the parameter ε in (1.1), so we consider the equation

$$u_{tt} + Au = f(u,t) \,. \tag{1.5}$$

We have used in [4] the Banach fixed point theorem. To get our results in this paper, we apply the Leray-Schauder fixed point theorem. For this reason, we need more precision condition than (1.2), see (2.2) below. We also study the resonant case when $0 < \dim\ker A < \infty$. We derive a Landesman-Lazer type result [7]. Finally, we present a forced beam equation as an example.

We secondly investigate more correctly and thoroughly the condition (1.4) than in [4]. By using some results of the number theory [2, 3, 6], we derive several conditions for ω when (1.4) is either satisfied or not.