Funkcialaj Ekvacioj
Volume 44, No. 3, pp 471--486

Global Solvability and Asymptotic Behaviour of a Hyperbolic Problem With Acoustic Boundary Conditions

A. T. Cousin, C. L. Frota, and N. A. Larkin
(Universidade Estadual de Maringá, Brasil)

1. Introduction

This article presents a study of global solvability and asymptotic behaviour of solutions to the mixed problem for the nonlinear hyperbolic equation with nonlinear damping and acoustic boundary conditions:

$$u_{tt} - a(u)u_{xx} + g(u_t) = 0, \qquad \text{in } Q = (0,1)\times(0,T), \tag{1.1}$$

$$u(0,t) = 0, \qquad t \in (0,T), \tag{1.2}$$

$$-\rho u_t(1,t) = \delta_{tt}(t) + c_1\delta_t(t) + c_0\delta(t), \qquad t \in (0,T), \tag{1.3}$$

$$u_x(1,t) = \delta_t(t), \qquad t \in (0,T), \tag{1.4}$$

$$u(x,0) = u_0(x)\ ,\ u_t(x,0) = u_1(x),\ x \in (0,1), \quad \delta(0) = \delta_0; \tag{1.5}$$

where ρ, T, c_0, c_1 are positive constants, $a : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are differentiable functions.

In [3], J. T. Beale and S. I. Rosencrans considered acoustic boundary conditions for a gas, undergoing small irrotacional pertubations from the rest, in a domain $\Omega \subset \mathbb{R}^3$ with a smooth compact boundary $\partial\Omega = \Gamma$. They assumed that the boundary Γ is nonporous and locally reacting in the sense that wave motion along the boundary is negligible. Therefore, if $u(x,t)$ is the velocity potential and $\delta(x,t)$ denotes the normal displacement of a point $x \in \Gamma$ at time t, they must satisfy the equations:

$$u_{tt} - \Delta u = 0, \qquad \text{in} \quad \Omega\times(0,T); \tag{1.6}$$

$$\delta_{tt} - c_1\delta_t + c_0\delta = -\rho u_t, \qquad \text{on} \quad \Gamma\times(0,T); \tag{1.7}$$

$$\delta_t = \frac{\partial u}{\partial \nu}, \qquad \text{on} \quad \Gamma\times(0,T); \tag{1.8}$$

here ν is the unit outer normal, $c_1 = \frac{d}{m}$, $c_0 = \frac{k}{m}$, $\rho = \frac{\rho_0}{m}$; where m is the mass per unit area on the boundary, d is the resistivity of the boundary, k is the spring constant and ρ_0 is the unperturbed density of the gas.

The mixed problem for $(1.6) - (1.8)$ was considered by J. T. Beale which, using semigroup techniques, carried out a detailed analysis in bounded domain [1] and exterior domains [2]. Beale′s idea was to work with an equivalent initial value problem in the form $u_t = Au$ where A is an operator on a suitable Hilbert space H. For bounded domains, in [1], Beale gave a description of the spectrum of the operator A and proved that there is no uniform rate of decay for solutions to the mixed problem for $(1.6) - (1.8)$.

Recently, C. L. Frota and J. A. Goldstein [4] studied acoustic boundary conditions (1.7), (1.8) for the nonlinear Carrier equation with a nonlinear damping

$$u_{tt} - M(\int_\Omega u^2\, dx)\Delta u + \mid u_t \mid^\alpha u_t = 0. \tag{1.9}$$

The authors proved results on global solvability, uniqueness, regularity and continuous dependence of solutions on the parameters when the boundary Γ is made up of two disjoint pieces Γ_0, Γ_1, each having nonnempty interior. Acoustic boundary conditions $(1.7), (1.8)$ were imposed for $(x,t) \in \Gamma_1 \times (0,T)$ and the Dirichlet boundary condition $u(x,t) = 0$ was imposed for $(x,t) \in \Gamma_0 \times [0,T]$. Moreover, for dimensions $n = 2$ or $n = 3$, global solvability and uniqueness for the mixed problem for (1.9) with acoustic boundary conditions on the whole boundary Γ were stablished. This was obtained as a limit case when the positive measure of Γ_0 shrink to zero.

Global solvability and stability of the energy of the mixed problem for (1.9) with the Dirichlet boundary conditions was proved in [5].

In the present paper we consider the nonlinear wave equation (1.1) with local nonlinearities. The function $a(u)$ depends on a solution while the function $M(\int_\Omega u^2\, dx)$ in (1.9) depends on the L^2-norm of it. This difference makes the study of $(1.1) - (1.5)$ more complicated and we restrict it only to the one-dimensional case. Exponential and algebraic energy decay are proved under some appropriate assumptions on the damping $g(u_t)$ and when c_0 is sufficiently